

# PROBABILITY

Probability is the measure of uncertainty of various phenomenon, numerically. It can have positive value from 0 to 1.

The words 'probably', 'doubt', 'most probably', 'chances', etc., used in the statements above involve an element of uncertainty.

$$\text{Probability} = \frac{\text{no. of favorable outcome}}{\text{total no. of outcomes}}$$

Approach to Probability:

- i. **Statistical approach** : Observation & data collection
- ii. **Classical approach**: Only Equal probable events
- iii. **Axiomatic approach**: For real life events. It closely relates to set theory.

## 2. Random Experiments:

An experiment is called random experiment if it satisfies the following two conditions:

- (i) It has more than one possible outcome.
- (ii) It is not possible to predict the outcome in advance.

**Outcomes:** a possible result of a random experiment is called its outcome.

**Sample space:** Set of all possible outcomes of a random experiment is called sample space. It is denoted by the symbol S. Example: In Toss of a coin, Sample space is Head, Tail.

**Sample point:** Each element of the sample space is called a sample point. E.g. in toss of a coin, Head is a Sample point.

## 3. Event:

It is the set of favorable outcome.

Any subset E of a sample space S is called an event. E.g. Event of getting odd outcome in a throw of a die

**Occurrence of an event:** the event E of a sample space S is said to have occurred if the outcome  $\omega$  of the experiment is such that  $\omega \in E$ . If the outcome  $\omega$  is such that  $\omega \notin E$ , we say that the event E has not occurred.

## Types of Event

- i. Impossible and Sure Events
- ii. Simple Event
- iii. Compound Event

### Impossible and Sure Events:

The empty set  $\phi$  and the sample space S describe events. **Impossible event** is denoted by  $\phi$ , while the whole sample space, S, is called the **Sure Event**.

E.g. in rolling a die, impossible event is that number is more than 6 & Sure event is the event of getting number less than or equal to 6.

### Simple Event:

If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.

In a sample space containing n distinct elements, there are exactly n simple events.

E.g. in rolling a die, Simple event could be the event of getting 4.

### Compound Event:

If an event has more than one sample point, it is called a Compound event.

E.g. in rolling a die, Simple event could be the event of getting even number

## Algebra of Events

- i. Complementary Event
- ii. Event 'A or B'
- iii. Event 'A and B'
- iv. Event 'A but not B'

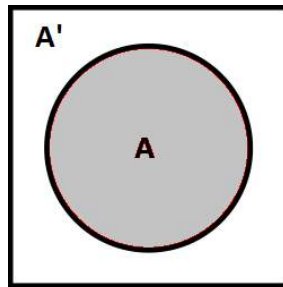
### Complementary Event

Complementary event to A = 'not A'

Example: If event A = Event of getting odd number in throw of a die, that is {1, 3, 5}

Then, Complementary event to A = Event of getting even number in throw of a die, that is {2, 4, 6}

$A' = \{\omega : \omega \in S \text{ and } \omega \notin A\} = S - A$  (Where S is the Sample Space)

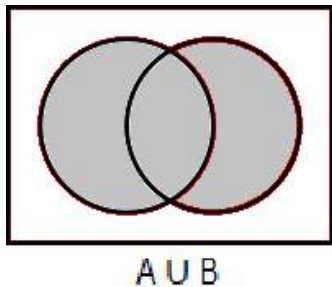


### Event (A or B)

Union of two sets A and B denoted by  $A \cup B$  contains all those elements which are either in A or in B or in both.

When the sets A and B are two events associated with a sample space, then ' $A \cup B$ ' is the event 'either A or B or both'. This event ' $A \cup B$ ' is also called 'A or B'.

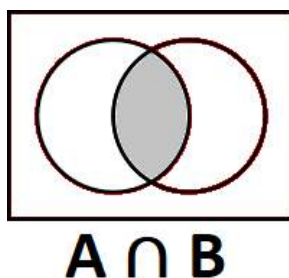
Event ' $A \cup B$ ' =  $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$ .



### Event 'A and B'

Intersection of two sets  $A \cap B$  is the set of those elements which are common to both A and B. i.e., which belong to both 'A and B'. If A and B are two events, then the set  $A \cap B$  denotes the event 'A and B'.

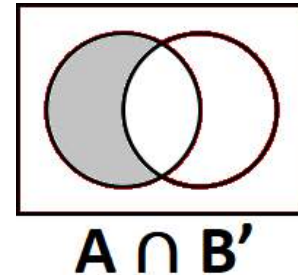
Thus,  $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$



### Event 'A but not B'

$A - B$  is the set of all those elements which are in A but not in B. Therefore, the set  $A - B$  may denote the event 'A but not B'.

$A - B = A \cap B'$



### Mutually exclusive events

Events A and B are called mutually exclusive events if occurrence of any one of them excludes occurrence of other event, i.e., if they cannot occur simultaneously.

Example: A die is thrown. Event A = All even outcome & event B = All odd outcome. Then A & B are mutually exclusive events, they cannot occur simultaneously.

Simple events of a sample space are always mutually exclusive.

### Exhaustive events

Lot of events that together forms sample space.

Example: A die is thrown. Event A = All even outcome & event B = All odd outcome. Even A & B together forms exhaustive events as it forms Sample Space.

### 4. Axiomatic Approach to Probability:

It is another way of describing probability. Here Axioms or rules are used.

Let S be sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval  $[0,1]$  satisfying the following axioms

- i) For any event E,  $P[E] \geq 0$
- ii)  $P[S] = 1$
- iii) If E and F are mutually exclusive events, then  $P(E \cup F) = P(E) + P(F)$ .

It follows from (iii) that  $P(\phi) = 0$ . Let  $F = \phi$  and  $E = \phi$  be two disjoint events,

$$\therefore P(E \cup \phi) = P(E) + P(\phi) \text{ or } P(E) = P(E) + P(\phi) \text{ i.e., } P(\phi) = 0$$

Let  $S$  be a sample space containing outcomes  $\omega_1, \omega_2, \dots, \omega_n$ , i.e.,  $S = \{\omega_1, \omega_2, \dots, \omega_n\}$  then

- i.  $0 \leq P(\omega_i) \leq 1$  for each  $\omega_i \in S$
- ii.  $P(\omega_1) + P(\omega_2) + \dots + P(\omega_n) = 1$
- iii. For any event  $E$ ,  $P(E) = \sum_{\omega_i \in A} P(\omega_i)$
- iv.  $P(\phi) = 0$

**Probabilities of equally likely outcomes:**

Let  $P(\omega_i) = p$ , for all  $\omega_i \in S$  where  $0 \leq p \leq 1$ ,

then  $p = \frac{1}{n}$  where  $n =$  number of elements.

Let  $S$  be a sample space and  $E$  be an event, such that  $n(S) = n$  and  $n(E) = m$ . If each outcome is equally likely, then it follows that

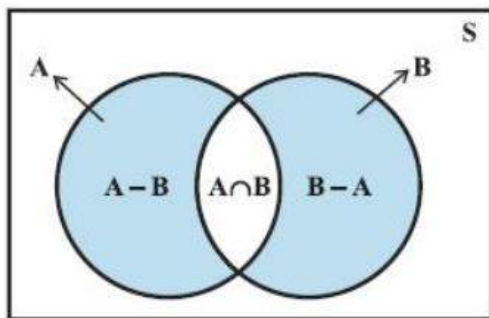
$$P(E) = \frac{m}{n} = \frac{\text{Number of Outcomes favourable to } E}{\text{Total possible outcomes}}$$

**Probability of the event 'A or B' :**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

**Probability of the event 'A and B'**

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$



**Probability of the event 'Not A'**

$$P(A') = P(\text{not } A) = 1 - P(A)$$